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ASW fusion on a PC

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THESIS

ASW FUSION ON A PC

by

Joelle J. Mann

June 2004

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Second Reader:	Roger Bacon

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LosCon, the software program developed for the author's thesis and tested at sea, is designed to help the ASW commander regain tactical control in a loss of submarine contact situation. Persistent detection and cueing in the battlespace depend on utilizing contact reports from a network of combatant platform and offboard sensors. LosCon, an extended Kalman filter-based program modeled after MTST (Maneuvering Target Statistical Tracker), can integrate the sensor network very efficiently. Kalman filtering is a method of recursively updating the position of an evading target and accuracy of that position using imperfect measurements. Lines of bearing to the contact with associated standard deviation bearing errors and positions with their standard deviation range errors are the measurements LosCon uses to generate an ellipse of the submarine's likely position or AOU (Area Of Uncertainty). LosCon will also generate an expanded AOU for any future time, allowing commanders to correctly estimate the size of the search area. The effectiveness of the sea shield concept depends on the ability of organic forces to deny the enemy tactical control of the battlespace area. Incorporating the information generated by LosCon would assist ASW commanders in maintaining undersea superiority.

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ASW FUSION ON A PC

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LosCon, the software program developed for the author's thesis and tested at sea, is designed to help the ASW commander regain tactical control in a loss of submarine contact situation. Persistent detection and cueing in the battlespace depend on utilizing contact reports from a network of combatant platform and offboard sensors. LosCon, an extended Kalman filter-based program modeled after MTST (Maneuvering Target Statistical Tracker), can integrate the sensor network very efficiently. Kalman filtering is a method of recursively updating the position of an evading target and accuracy of that position using imperfect measurements. Lines of bearing to the contact with associated standard deviation bearing errors and positions with their standard deviation range errors are the measurements LosCon uses to generate an ellipse of the submarine's likely position or AOU (Area Of Uncertainty). LosCon will also generate an expanded AOU for any future time, allowing commanders to correctly estimate the size of the search area. The effectiveness of the sea shield concept depends on the ability of organic forces to deny the enemy tactical control of the battlespace area. Incorporating the information generated by LosCon would assist ASW commanders in maintaining undersea superiority.

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EXECUTIVE SUMMARY
ASW FUSION ON A PC

Purpose:

LosCon, the software program developed in this thesis, is designed to assist the ASW commander regain tactical control in a loss of contact situation.

Background:

The changing strategic environment has de-emphasized traditional cold-war force-on-force tactics in favor of defeating the asymmetric submarine warfare threat. Enemy submarines pose a significant threat to the Sea Power 21 concept of operations. There are too many potential adversaries for the U.S. submarine force to control the entire undersea environment, yet the sea base and sea strike forces must retain their freedom to operate without the interference of enemy submarines. In the process of acquiring undersea supremacy, some periods of lost contact are to be expected. Minimizing these periods is vital to maintaining undersea superiority.

Discussion:

Persistent detection and cueing in the battlespace depend on utilizing contact reports from a network of combatant platform and offboard sensors. LosCon, an extended Kalman filter based program modeled after MTST (Maneuvering Target Statistical Tracker), can effectively integrate the sensor network. Kalman filtering is a method of recursively updating the position of an evading target and accuracy of that position using imperfect measurements. Lines of bearing to the contact with associated standard deviation bearing errors and positions with their standard deviation range errors are the measurements LosCon uses to generate an ellipse of the submarine's likely position or AOU (Area Of Uncertainty). The AOU is updated with each new contact report. LosCon will also generate an expanded AOU for any future time. This allows commanders to estimate the size of the area that would need to be searched after an extended period of lost contact.

LosCon was developed following the observation of an ASW exercise at sea with the JCS Strike Group in the Southern California Operating Areas (SOCAL). It was then tested at sea during an exercise with the 2nd Expeditionary Strike Group. Following a loss of contact, a search plan was designed based on the AOU generated by LosCon for the time when maritime patrol craft would arrive on station. The aircraft found a surfaced submarine near the center of the AOU.

Conclusions:

The effectiveness of the sea shield concept depends on the ability of organic forces to deny the enemy tactical control of the battlespace area. Minimizing periods of lost contact is vital to maintaining undersea superiority. Incorporating the information generated by LosCon would assist ASW commanders in maintaining undersea superiority. A further evaluation of LosCon for operational utilization should be considered.

I. ANTI-SUBMARINE WARFARE

A. SUBMARINE THREAT

The twenty-first century has seen the rise of a variety of nontraditional threats including transnational terrorist organizations and the rogue nation states that are their traditional sponsors. Several of these states possess sophisticated asymmetric naval forces, including submarines. The changing strategic environment has de-emphasized traditional cold-war force-on-force tactics in favor of defeating the asymmetric submarine warfare threat. Enemy submarines pose a significant threat to the Sea Power 21 concept of operations. There are too many potential adversaries for the U.S. submarine force to control the entire undersea environment, yet the sea base and sea strike forces must retain their freedom to operate without the interference of enemy submarines. In the process of acquiring and maintaining undersea supremacy, some periods of lost contact are to be expected.[1] "Technology advances and exploitation permit the pursuit of additional approaches to ASW not previously available [and] sophisticated decision support systems […] will result in increased probability of detection," according to Chief of Naval Operations Admiral Clark's ASW CONOPS.[2] The software program, LosCon, is designed to help war-fighters quickly regain tactical control in a loss of contact situation.

Regaining tactical control is a complex process. When loss of contact occurs, individual units will attempt to relocate the enemy submarine, but the needs associated with avoiding counter detection make the problem difficult.[3] Tracking a particular enemy submarine for an extended period of time or over a wide area increases the difficulty of the problem. To assist in the decisions on how to conduct the search, LosCon uses all the information available to generate an AOU (Area Of Uncertainty) for the current time and any time in the future when assets may become available.

[1] Clark, Vern. *Anti-Submarine Warfare Concept of Operations (Draft 26 April 2004)*.

[2] Ibid, 7.

[3] U.S. Navy Publication. *NTTP 3-21.23 Submarine Tracking Manual.* (Washington, D.C.: U.S. GPO), 83.

B. NATURE OF ASW

The U.S. Navy's current ASW (Anti-Submarine Warfare) policy involves combining information from a wide variety of platform-based sensors, fixed land-based sensors, and intelligence sources.[4] Anti-Submarine Warfare is often referred to as an art in deference to the many educated estimates made to track a submarine over an extended period.

Effective submarine tracking involves a detailed understanding of the local ocean environment, the type of submarine threat, and the available ASW assets.[5] Most assets have multiple warfare roles, the requirements of which may not often be compatible with optimal ASW positioning. Extended periods without a credible enemy submarine contact are not conducive to maintaining an ASW focus when assets are also involved in other more active warfare roles.[6]

Conversely, once there is evidence of an enemy submarine in the area, especially following a successful attack on U.S. forces, there is a "natural tendency to confirm any unexplained sighting as a submarine and to assume that any periscope or submarine detection is an enemy submarine."[7] This results in a large quantity of false contact reports when there is no submarine present, and the possibility of friendly fire incidents and collateral damage when surface or air forces mistakenly identify ocean life, friendly or neutral submarines as enemy submarines. In his biography Admiral Sandy Woodward, the United Kingdom's battle group commander during the Falklands War, commented on frequent "submarine contacts" that turned out to actually be whales.[8] In WWII, U.S. surface forces sank two U.S. submarines after mistaking them for enemy combatants. The number of false contact reports associated with submarine tracking remains high, and must be considered in any search algorithm designed to locate enemy submarines.

---

[4] U.S. Navy Publication. *NWP 3-21 Navy ASW.* (Washington, D.C.: U.S. GPO), 29.

[5] Ibid.

[6] Ibid, 34-35.

[7] Ibid, 35-36.

[8] Woodward, John. *One Hundred Days: the Memoirs of the Falklands Battle Group Commander.* (Annapolis: Naval Institute Press, 1992), 96-97.

A search for a submarine often begins with theater and or national intelligence. Local oceanographic information must then be analyzed to determine the best way to prosecute the search considering the assets available.[9]

> ASW assets encompass a broad range of systems that can provide a serious threat to an enemy submarine while it is submerged. ASW capable U.S. assets include:
>
> - National sensor systems
> - IUSS (Integrated Underwater Surveillance System)
> - ASW capable surface ships
> - ASW fixed wing aircraft (P-3, S-3)
> - ASW helicopters (SH-60)
> - Submarines.[10]

All of these assets have non-ASW roles as well. Thus it is not surprising that, whether attempting to prosecute a real-world or exercise problem, ASW forces experience greatest difficulty maintaining tactical control when the enemy submarine has crossed into a new area of responsibility.[11] It may not be possible for the new commander to assign sufficient assets to tracking the enemy submarine.

C. OVERVIEW OF THESIS

The effectiveness of the sea shield concept depends on the ability of organic forces to deny enemies tactical control in the areas occupied by sea strike groups and sea bases. Control of the sea in the Mahanian sense is not feasible without sufficient U.S. submarines to match potential enemy submarines one for one. Loss of contact on a hostile or potentially hostile submarine threatens the US force's ability to successfully conduct their missions.

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[9] U.S. Navy Publication. *NWP 3-21 Navy ASW.* (Washington, D.C.: U.S. GPO), 30.

[10] Ibid, 34-35.

[11] Ibid, 29-30.

LosCon is designed to assist in regaining tactical control in a loss of contact situation. The ellipse of the submarine's probable location, its AOU, can be used to move vulnerable assets away from the threat and to focus searchers efforts to most effectively regain contact.

LosCon's development included observing an ASW exercise conducted by the JCS Carrier Strike Group and testing of the program in an ASW exercise with the 2nd Expeditionary Strike Group. LosCon's mathematical core is an iterated extended Kalman filter employing the MTST (Maneuvering Target Statistical Tracker).[12] The following chapters will explain iterated extended Kalman filters, their application in LosCon, and LosCon's attributes, and potential ways in which LosCon might be expanded.

[12] Wagner, Daniel H., "Naval Tactical Decision Aids," *Military Operations Research Lecture Notes*, (September 1989).

II. THEORECTICAL BASIS OF LOSCON

A. ITERATED EXTENDED KALMAN FILTERING THEORY

Kalman filtering, an algorithm frequently used in localization and tracking aids, is a method of recursively updating the mean of a system based on a series of measurements. The system is modeled in two parts: measurement and movement.

In LosCon, the target's true state X consists of two location components and two velocity components as shown.

$$X = \begin{bmatrix} \text{Cartesian Coordinate from Longitude} \\ \text{Cartesian Coordinate from Latitude} \\ \text{Velocity in the East-West Direction} \\ \text{Velocity in the North-South Direction} \end{bmatrix}$$

The true state cannot be known exactly, so it is treated as a random variable. Specifically, X is assumed to be a multivariate normal random variable with mean μ and covariance matrix Σ, symbolically X~N(μ,Σ). The symbol ~ will frequently be used to indicate the probability distribution shown on the right-hand side. The Kalman filter repeatedly revises μ and Σ to account for the passage of time (X changes with time), as well as for measurements.

The new state of the system after the passage of time, X', is modeled by the product of the movement matrix ϕ and the old state X, summed with the error distribution associated with movement, $W : N(\mu_W, Q)$.[13]

<u>Movement Model:</u>

$$X' = \phi X + W$$

It follows that $X' : N(\mu', \Sigma')$, where $\mu' = \phi\mu + \mu_W$ and $\Sigma' = \phi \Sigma \phi^T + Q$. If ϕ depends on X, the Kalman filter is *extended*.

[13] Washburn, A. "A Short Introduction to Kalman Filters," (NPS 2004), 1-4.

The measurement *Z* is the product of a measurement matrix H and the state of the system *X*, summed with the error distribution associated with that type of measurement, $V: N(\mu_V, R)$.[14]

Measurement Model:

$Z = HX + V$

When Z is given, it follows that $X: N(\hat{\mu}, \hat{\Sigma})$, where $\hat{\mu} = \mu + K(Z - \mu_V - H\mu)$ and $\hat{\Sigma} = (I - KH)\Sigma$. Here $K = \Sigma H^T (H \Sigma H^T + R)^{-1}$ is the Kalman gain. If H depends on *X*, as it does in LosCon, the Kalman filter is *extended*.

Kalman gain is the amount by which the old estimate for the state of the system is changed by introducing a new measurement. The shock, $S = Z - \mu_v - H\mu$, is the difference between the measurement and what the Kalman filter expected the measurement to be based on all past measurements. Recall that $\hat{\mu} = \mu + K\left(Z - \mu_v - H\mu\right)$; $\hat{\mu}$ is updated by summing the product of the Kalman gain and the shock with the previous μ.[15] Dimensionless shock for a specific measurement i, $DS_i \equiv S_i^T \left(H_i \Sigma_i H_i^T + R\right)^{-1} S_i$, can be thought of as normalized shock, and it is used to tell when the shock, S_i, is too large.[16]

Recall that H is the measurement matrix that defines the measurement's relation to the state, and ϕ is the movement matrix that defines how the state changes with time. If either H or ϕ depends on X, the respective measurement or movement model becomes nonlinear. Kalman filtering is a linear process, so nonlinearity introduces a complication. When either of the models must be linearized, the process is referred to as an extended Kalman filter.

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14 Ibid.

15 Balakrishnan, A.V., *Kalman Filtering Theory,* (New York: Springer-Verlag, 1984), 76-96.

16 Washburn, A. "A Short Introduction to Kalman Filters," (NPS 2004).

Iteration is useful in extended Kalman filters for minimizing error.[17] One such case is when measurements can be bearings-only as they are in LosCon. For a bearings-only measurement, H depends on X. Greater detail for this case will be provided in section "B: Implementation in Excel." Recall that in the measurement model, $\hat{\mu} = \mu + K(Z - \mu_V - H\mu)$. Some value must be entered for the initial estimate of the mean at the time of the first measurement. The value of the state of the system at the time of the first measurement is referred to as $X_{z=1} : N(\mu_{z=1}, \Sigma_{z=1})$ where the index z refers to the time associated with the Z^{th} measurement. Let i be the index of the iteration, which is distinct from the measurement's time index z. The mean of the state, $\hat{\mu}$, can be recalculated repeatedly as shown: $\mu_i = \mu_{i-1} + K(Z - \mu_V - H_{i-1}\mu_{i-1})$ until $\mu_{i-1} \approx \mu_i$. The iteration converges when $\mu_{i-1} \approx \mu_i$. Let n refer to the number of iterations at which convergence occurs. Iteration is done to minimize the error introduced by the initial estimate of the mean, $\mu_{z=1,\, i=1}$. When iteration results in convergence, $\mu_{z=1,\, i=1}$ is replaced by $\mu_{z=1,\, i=n}$.

The mean of the state does not always converge, because in some cases the linearized measurement matrix, H, will cause the Kalman gain, $K = \sum H^T (H \sum H^T + R)^{-1}$, to update $\hat{\mu}$ in the wrong direction. Divergence during the iteration process can occur when the initial estimate of X, $\mu_{z=1,\, i=1}$, is poor or the measurements, Z, have large errors .

The iterated extended Kalman filter is a useful tool in spite of the need to linearize non-linear models, because iteration can minimize the error introduced in the linearization process. The potential for divergence during iteration is tolerable because the filtering process can be restarted easily by selecting a new initial estimate. Divergence also serves as a warning for the existence of bad data.

---

[17] Bell, Bradley M., and Fredrick W. Cathey. "The Iterated Kalman Filter Update as a Gauss-Newton Method," *IEEE Transactions on Automatic Control* 38, no. 2 (February 1993): 294-297.

B. IMPLEMENTATION IN EXCEL

LosCon is modeled after MTST, the program with which Daniel H. Wagner and Associates pioneered using Kalman filters to track submarines in 1980.[18] An explanation of LosCon's iterated extended Kalman filter will follow. When naval terminology with similar meanings to the precise mathematical terminology exists, it will be included.

Recall that

$$\mu = E(X) = \begin{bmatrix} \text{Estimated Mean Cartesian Coordinate from Longitude} \\ \text{Estimated Mean Cartesian Coordinate from Latitude} \\ \text{Estimated Mean Velocity in the East-West Direction} \\ \text{Estimated Mean Velocity in the North-South Direction} \end{bmatrix} = \begin{bmatrix} \mu_{lon} \\ \mu_{lat} \\ \mu_{\text{E-W Vel}} \\ \mu_{\text{N-S Vel}} \end{bmatrix}$$

The Kalman filter begins with an estimate of $\mu_{z=1,\, i=1}$ and a covariance matrix, $\sum_{z=1}$, estimating the accuracy of that mean.[19] LosCon uses an extended Kalman filter, because some of the measurements are bearings only. Recall that for extended Kalman filters iterating $\mu_{z=1,\, i=1}$ can minimize error.[20] The initial covariance matrix is not iterated in LosCon because the movement model is linear. Since $\sum_{z=1}$ is not affected by the iteration of μ, $\sum_{z=1}$ will be written as $\sum_1$. The initial covariance matrix has the default value shown below.

$$\sum_1 = \begin{bmatrix} 100,000 & 0 & 0 & 0 \\ 0 & 100,000 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}.$$

See Appendix B: Visual Basic Code to modify the initialization of $\sum_1$.

LosCon generates AOUs based on the position components of μ and the standard deviations from $\sum$.[21]

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18 Wagner, Daniel H., "Naval Tactical Decision Aids," *Military Operations Research Lecture Notes*, (September 1989).

19 Balakrishnan, A.V., *Kalman Filtering Theory,* (New York: Springer-Verlag, 1984), 76-96.

20 Kerr, Thomas H.. "Streamlining Measurement Iteration for EKF Target Tracking," *IEEE Transactions on Aerospace and Electronic Systems* 27, no. 2 (March 1991): 408-421.

21 Washburn, A. "A Short Introduction to Kalman Filters," (NPS 2004), 4.

A measurement in LosCon corresponds to a contact on a submarine. Recall that the measurement matrix, H, is a description of what kind of information the measurements contain. The value of H depends on the type of measurement. Measurements in LosCon are of two types.

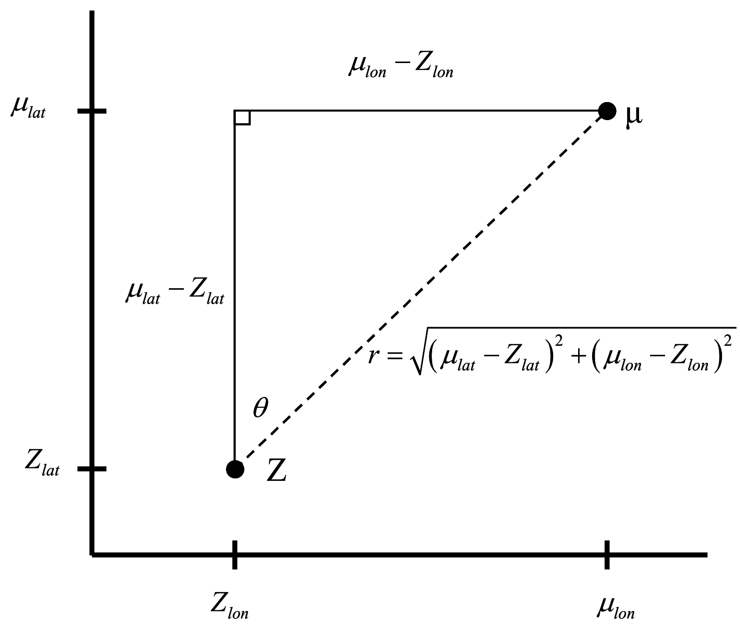
1) Position measurements consist of the latitude and longitude of the target, a single standard deviation error in nautical miles, and the time of the contact. For these measurements, H is a fixed value.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

2) Bearing measurements consist of a bearing taker's latitude and longitude, a bearing on a target, the single standard deviation bearing error, and the time of the contact. H varies based on the state.

$$H = \begin{bmatrix} \dfrac{\mu_{lat} - Z_{lat}}{\left(\mu_{lat} - Z_{lat}\right)^2 + \left(\mu_{lon} - Z_{lon}\right)^2} & \dfrac{-\left(\mu_{lon} - Z_{lon}\right)}{\left(\mu_{lat} - Z_{lat}\right)^2 + \left(\mu_{lon} - Z_{lon}\right)^2} & 0 & 0 \end{bmatrix}$$

where μ_{lat} is the "y" component of the initial estimate which corresponds to latitude, μ_{lon} is the "x" component of the initial estimate which corresponds to longitude, Z_{lat} is the bearing taker's latitude converted into Cartesian coordinates, and Z_{lon} is the bearing taker's longitude converted into Cartesian coordinates. Figure 1 shows the relationship between the four terms.



$$\frac{\cos\theta}{r} = \frac{\mu_{lat} - Z_{lat}}{\left(\mu_{lat} - Z_{lat}\right)^2 + \left(\mu_{lon} - Z_{lon}\right)^2}$$

$$\frac{-\sin\theta}{r} = \frac{-\left(\mu_{lon} - Z_{lon}\right)}{\left(\mu_{lat} - Z_{lat}\right)^2 + \left(\mu_{lon} - Z_{lon}\right)^2}$$

Figure 1: The terms in the measurement matrix of a bearing measurement are shown in trigonometric form.

Since H changes based on the μ_{lat} and μ_{lon} components of the state for bearing measurements and H is a function of $\mu_{Z=1} = \left[\mu_{lon} \quad \mu_{lat} \quad \mu_{\textit{E-W Velocity}} \quad \mu_{\textit{N-S Velocity}}\right]^T$, recursively calculating $\mu_{Z=1}$ can minimize the error introduced in the extended Kalman filter by the initial estimate.[22] Recall that in the course of this iteration it is also possible for the values of μ to diverge. This occurs when the extended Kalman filter updates μ in the wrong direction.

[22] Kerr, Thomas H.. "Streamlining Measurement Iteration for EKF Target Tracking," *IEEE Transactions on Aerospace and Electronic Systems* 27, no. 2 (March 1991): 408-421.

After measurements are entered, the filter should be iterated. If μ converges within the first several iterations, an AOU is returned, and LosCon has performed as desired. If that does not happen, shock can be used to help diagnose what went wrong. Recall that the shock, $S = Z - \mu_v - H\mu$, is the difference between the measurement and what the Kalman filter expected the measurement to be, based on all past measurements. Dimensionless shock, $DS_i \equiv S_i^T \left(H_i \sum_i H_i^T + R\right)^{-1} S_i$, is used to tell when the shock, S_i, is too large.[23] High dimensionless shocks will occur when measurements with large errors are entered in LosCon , or when the initial estimate for the location and velocity of a target submarine is poor. The challenge of dealing with high dimensionless shocks is that while they may indicate that the new contact report is false, they may instead mean that the last few reports were false and that the current contact report is true.

ASW remains an art. LosCon is a tool to help the war-fighter determine which contact reports are true and where the submarine would be if contact reports in a particular group were assumed to be true. The toggle switch in LosCon allows the war-fighter to quickly check multiple groups of contact reports against each other to find which sets could be accurate.

23 Washburn, A. "A Short Introduction to Kalman Filters," (NPS 2004).

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III. TACTICAL UTILITY OF LOSCON

A. OBJECTIVE

LosCon utilizes the well-understood mathematical principles of Kalman filtering to make tracking submarines in a localized area easier. Kalman filtering is commonly used for problems that include significant error factors. In the submarine-tracking problem, the error types are two-fold:

(1) A large proportion of the contact reports can be assumed to be false, although exactly which ones are which is not immediately clear. Sometimes later observations reveal some reports to be impossible, or at the conclusion of an exercise the submarine's track can be compared to the contact reports.

(2) Each true contact report also includes a certain, non-constant amount of error. A sonar contact report will have an associated bearing error. A periscope sighting will have bearing and range error. All other contact reports such as those associated with sonobuoy fields or IUSS will have their own errors as well.

LosCon minimizes the effects of false contact reports and can use the error associated with true contact reports to create an AOU for the location of the enemy submarine.

The most effective method for false contact reports is to ignore them. Since the operator cannot know for certain which contact reports are false, LosCon includes a toggle switch that allows the operator to selectively ignore and include contacts, running the program multiple ways to visually understand each contact reports' effect on the AOU.

Operators are faced with a complex analytical problem after losing contact. They have a number of old bearings on the contact which are assumed to be correct, certainly contain some error, and may in fact not even be all true. Based on the contact reports that the operator has selected as true, an AOU is generated by LosCon. As new information about the location of the submarine is entered, the AOU changes and the operator can

maintain a clear tactical picture by considering all the information in one view screen. In the case where assets need to fulfill a non-ASW role for a certain time period, the operator can enter that future time to see the expanded AOU for that future time based the assumption that no new contact reports will be made in the intervening hours.

LosCon can track up to three submarines. Initial contact may not include sufficient information to correctly distinguish the enemy submarines, but the input data remains on screen and accessible throughout the exercise so that an operator can redesignate bearing and position reports as contacts on a different submarine or false contacts at any time.

B. DEVELOPMENT AND TESTING

LosCon was designed for use in a strike group's ASW command center to track and reacquire submarine contacts. During the early development stage, the author went to sea on the USS JOHN C. STENNIS (CVN 74), a Nimitz class aircraft carrier, to observe an ASW battle problem that the carrier strike group was conducting as part of her pre-deployment exercises. While the carrier has too much ship noise to be an effective anti-submarine warfare platform, the ASW commander is on board the carrier, and so all the contact reports are sent back to the carrier.

The ASW module on the carrier proved to be crowded with computers and people. The ASWO (ASW Officer) was equipped with computer systems predicting the group's sonar coverage, and also with chat connections to many of the other platforms in the group. Once contact was made, it was plotted on a traditional chart and the ASWO was left to estimate where the submarine might be next. When no details about the accuracy of the contact report position were mentioned, sometimes a quarter was centered on the plot and used to draw a circle around the point. Without the information needed to make a more educated estimate of the error there was no reason to do otherwise.

The exercise led to several observations about the on board ASW operational analyses process in use. The ASWO was not getting all the information available. Physical space in the ASW module was limited, and there was not a computer that could

be solely devoted to a new program. Most importantly, when loss of contact occurred, the ASWO did not have tools to assist in refining the situation for the purpose of regaining contact.

As such, during the next several months of development, LosCon's design was refined to optimize its practicality. LosCon was subsequently tested onboard USS MOBILE BAY. The Expeditionary Strike Group had an ASW battle problem during its final battle problem in which a U.S. nuclear submarine played the role of a diesel submarine. During the ASW component of the two-day battle problem, the search platforms made contact with the submarine and maintained contact for a few hours early in the problem. The searchers lost contact for several hours but had maritime patrol aircraft coming on station with sonobouy laying capabilities. Based on LosCon's predicted AOU for the time the aircraft would arrive, a search pattern was laid out. Less than a half hour into the search, the aircraft located the submarine near the center of the predicted AOU.

C. AOU GENERATION

Based on past contact reports, LosCon seeks to generate AOUs containing a submarine's most likely position at a given time. A series of figures will be used to show the connection between the standard deviation errors associated with each contact report type and the generated AOUs.

Figure 2 shows hypothetical concurrent contact reports to demonstrate the use of two bearing contact reports with their associated standard deviation bearing errors to generate a set of current AOUs. Elongated along the direction of the lines of bearing, the AOUs shown include the target submarine with probabilities of 39%, 86.5%, and 99%, which correspond to one, two, and three standard deviations respectively. The standard, which LosCon uses, is the 86.5% probability AOU.

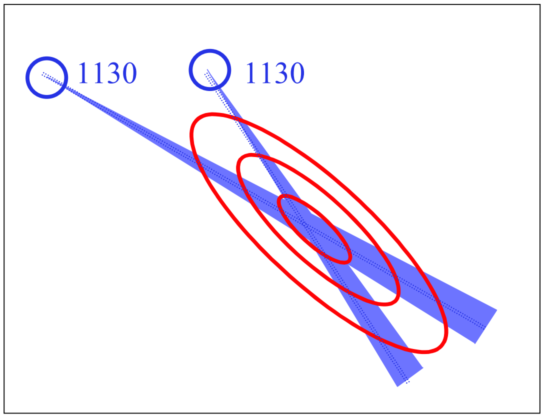

Figure 2: Lines of bearing with associated errors and the resulting AOUs.

If instead of lines of bearing, the contact reports on the target submarine were position reports from another unit, the AOUs would be circular as shown in Figure 3.

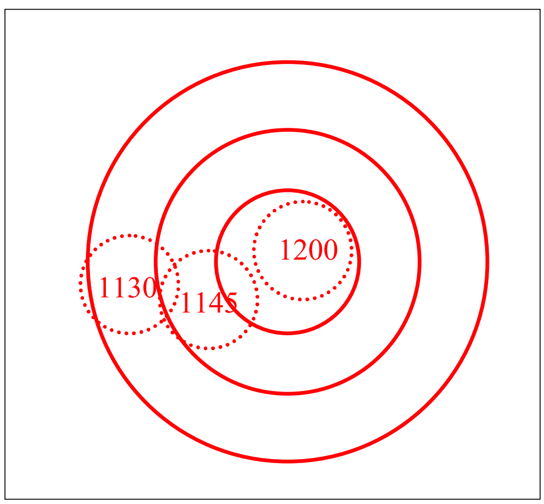

Figure 3: AOUs generated using position reports with circularly symmetric errors.

Generally, by the time a submarine has been tracked for a while and then lost, there are multiple types of contact reports for the target, some of which are considered false, and the generation of an AOU becomes significantly more complicated. Figure 4 shows such a scenario.

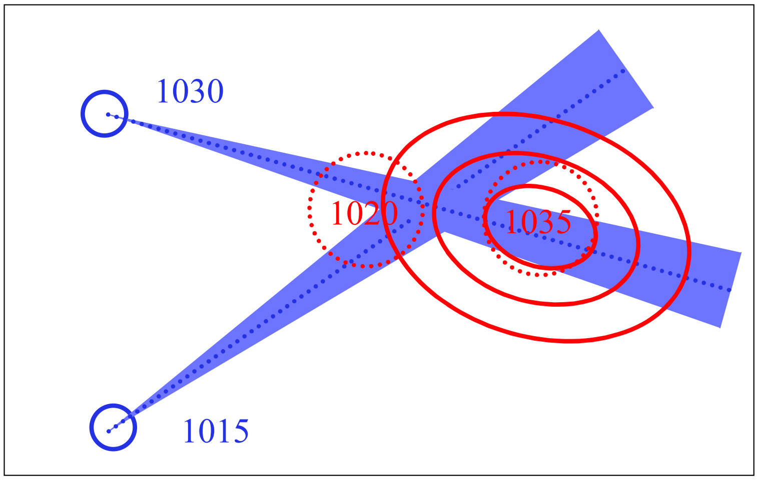

Figure 4: Multiple lines of bearing and positions with associated errors are combined to create 39%, 86.5%, and 99% AOUs.

Once a contact is lost, LosCon can continue to update the AOUs for the current time or even some future time. Figure 5 shows the 86.5% AOU at the time of contact loss. In Figure 6, LosCon uses all the information on the contact's position used in Figure 5 to create the AOU where search efforts can expect to regain contact for a future time.

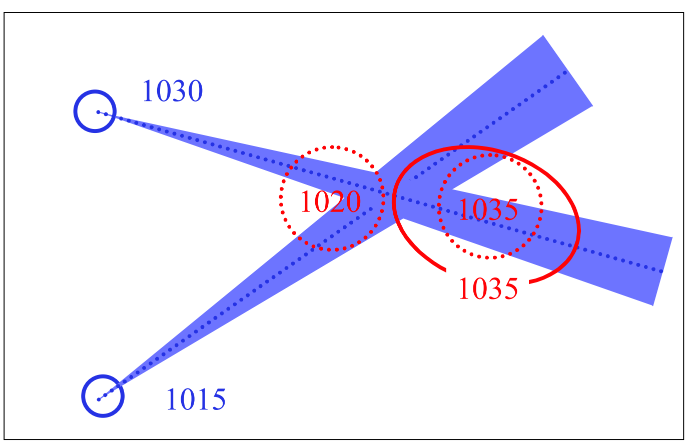

Figure 5: A standard (86.5%) AOU at the time of last contact.

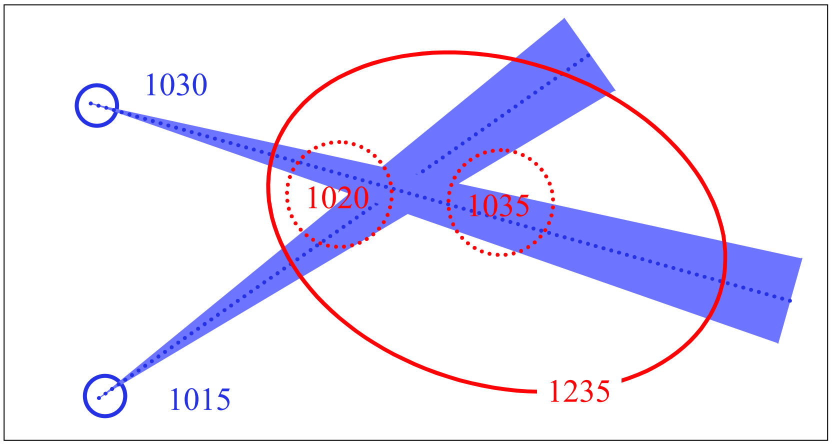

Figure 6: The same (86.5%) AOU is enlarged two hours later with no further contacts.

The method by which LosCon converts contact reports into AOUs, such as shown in the last five figures, is an iterated extended Kalman filter.

D. LIMITATIONS AND STRENGTHS

There are limitations inherent in using LosCon's Kalman filtering search process. The most significant limitation is the need for an initial estimate for where the enemy submarine could be. As explained in the previous chapter, this cannot be avoided when using Kalman filters. A bad initial estimate for the enemy submarine starting point at time zero can result in LosCon failing to return a usable result.

A lesser limitation is the need for an estimate of the target submarine's long-term average velocity, μ_w. This is the average of all courses and speeds a submarine could be reasonably expected to follow. When all courses and all potential speeds are equally likely, the long-term average velocity is zero. Absent any other information, zero is the logical estimate. This is the default setting in LosCon. An example for which this would be a correct assumption is a submarine attempting to target a sea base. The submarine would likely move around the area at varying courses and speeds, but no particular direction is more favored than another. A submarine traveling from one port to another should not have a zero estimate. While it will vary course and speed, it is going

somewhere and thus must have an overall non-zero velocity. If no credible intelligence is available concerning a submarine's intentions, a zero estimate for long-term average velocity is best. Zero is the average of all possibilities, and the effects of non-zero velocities on the center point of the AOU are small compared to the size of the AOU. To change LosCon to allow the long-term average velocity to be varied from zero, refer to the comments in Appendix B: Visual Basic Code.

The openness of LosCon's database makes these limitations tolerable. At any time the operator can select a new initial estimate for the location and velocity of a particular target submarine without having to reenter anything else. Similarly, each contact report can be toggled false or true at any point, and each contact report can be marked as a contact report on submarine one, two or three. All the report data entered remains clearly visible to the operator on a master spreadsheet. The initial estimates for the location and velocity of each submarine at time zero along with the future time at which an AOU is desired are located on supporting spreadsheets for each potential enemy submarine.

A number of LosCon's attributes were designed specifically for ease of use. For example the program file, LosCon.xls, is small enough to fit on an 1.44MB floppy disk. Microsoft Excel, software already installed on most ships' computers, was chosen as the interface. The nature of the Kalman filter algorithm makes the program operation low memory so the system requirements are not an issue. LosCon can be left as an open window in the background as the computer was used to do other operations. The database remains open to changes. Based on the limitations of Excel and a desire to keep LosCon floppy disk sized, the number of submarines was limited to three. To account for the localization math being done before the contact information was sent to the ASW command, an option of entering the contact reports as positions with circularly symmetric standard deviations in nautical miles was included. The expected contact report form of a platform location, bearing to the target, and bearing error was kept as an option.

In summary, the Kalman filter algorithm recursively calculates an AOU for the target submarine using repeated measurements of the submarine's location. An initial state must be estimated, but the associated covariance matrix is set by default to be very

large, and that estimate does not affect the AOU appreciably after two or more contact reports are entered and the filter goes through sufficient iteration to converge.

IV. FUTURE DEVELOPMENT

A. SUGGESTIONS FOR PROGRAM IMPROVEMENT

LosCon's interface is a simple Microsoft Excel spreadsheet using the graphical plug-in to generate visual representations of the mathematical program results. These built-in graphical capabilities do not use standard Navy icons. An updated version could use a different interface to allow for surface ships, submarines, and aircraft to be displayed in standard icons. While creating an independent interface program may be simpler, keeping the program essentially an Excel file allows it to be easily burned on CDs and used as a source of additional information during tactical engagements and exercises.

In some ways, LosCon may be too automated. Making it possible for the operator to change some built-in variables from their default settings could make LosCon more useful. Changing the root mean square velocity to correspond to specific submarine types would increase the usefulness of LosCon. Tables with cruising speeds for each type of submarine could be added to the program, so an operator could input the submarine type and LosCon would call the appropriate root mean square velocity for use in calculations. The default long-term average velocity for the enemy submarine could also be made variable to allow LosCon to be used for tracking transiting submarines.

LosCon could also be made easier to use if parts of it were more automated. The iteration process to find an initial estimate that converges could be built into the code so that it was accomplished with one click rather than many. The re-initialization of the initial estimate with every new contact report could also be built in. Ideally, one button press would start the iteration process to generate a converging initial estimate based on the first few contact reports, and then apply all the additional measurements to that estimate to generate an AOU at the time of the last contact report.

For LosCon to be used optimally, the single standard deviation error needs to be included in the program input. Standard contact report formats used on surface ships do not require inclusion of the standard deviations of bearing error or range error. The errors

are not so random that standard deviations are unattainable. Towed arrays, for example, have varying errors based on the directionality. The exact numbers would vary to some small extent but the test values generated when the system was first installed could be used. Error, even when it is reported, is not in a standard format such as a standard deviation or variance. It is important that the single standard deviation values inputted in LosCon be more than operator estimates. If standardization of error reports were sufficiently emphasized for good records to be kept of current and developing systems, single standard deviation bearing errors in degrees based on source and directionality could be tabulated and included in the next version of LosCon.

B. TACTICAL DECISION AID DEVELOPMENT OPTIONS

The usefulness of LosCon is limited, because it does not deal with ocean acoustics. A number of questions involving both oceanographic and probabilistic data remain to be answered. Where inside the LosCon-generated AOU are operators able to pick up sonar signals, for instance? Where should surface platforms reposition themselves to form sonar nets with onboard sensors and towed? Where should maritime patrol craft drop sonobuoy fields to optimize the chances of reacquiring the lost contact? An ideal ASW tactical decision aid package would include both oceanographic and probabilistic data.

The properties of the ocean which affect sound propagation and thus sonar effectiveness have been well studied and well modeled by a number of software packages—perhaps the most well known is IMAT.[24] These programs deal in the oceanographic issues for optimizing the volume of water with can be observed through sonar. These are traditionally extremely data-intensive computations that require dedicated hardware. It is still a data-intensive process, but mathematical and coding efforts have slimmed the program enough to create a useable shipboard oceanographic modeling program in PC IMAT.

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24 Czech, Carl. U.S. Navy Publication, *NPRDC-TR-98-2 The Interactive Multisensor Analysis Training (IMAT) System: an Evaluation of Airborne Acoustic Mission Course*, (San Diego, CA: GPO, 1998).

The nature of Kalman filtering in which data is repeatedly overwritten makes memory demands of generating the AOU for a contact extremely small. LosCon fits on a floppy disk even with contact reports. If LosCon were compiled with a new version of PC IMAT or a similar oceanographic program, LosCon would add negligible amounts to system memory requirements and file size. The value of the final product, however, would be greatly increased.

The ASW commander could look at a single screen displaying the convergence zone rings from combatant platforms and offboard sensors along with the enemy submarine's area of uncertainty. The commander could then overlay sonobuoy patterns and reposition assets over the AOU to maximize the likelihood of reacquiring contact.

Such an ASW package could easily include the following displays:

- Display the effects of oceanographic properties on sonar equipment. (IMAT) [25]
- Display areas of uncertainty for multiple contact reports. (LosCon)
- Display AOUs at a future time t if no new contact reports are made. (LosCon)

Once probabilistic target location and oceanographic properties are combined, a myriad of extremely useful options become possible. Given a time for the drop and the number of sonobuoys available, one could generate the optimal sensor placement. That sonobuoys could only be dropped in a fixed number of lines could be included in the constraints along with an expected variance to the exact placement plan. Optimal sensor placement could be generated, given additional limits such as not allowing a carrier to change course or speed during a fixed period due to flight operations. Loss of contact itself could become a tracking tool if the combination of a submarine's probabilistic location and the ocean environment allowed the war-fighter to predict that the AOU would pass out of the convergence zone.

LosCon is a useful and proven program which can help war-fighters regain tactical control in a loss of contact situation. Persistent detection and cueing, one of the fundamental principles of Chief of Naval Operations Admiral Clark's ASW Concept of Operations, depend on utilizing contact reports from a network of combatant platforms

[25] Czech, Carl. U.S. Navy Publication, *NPRDC-TR-98-2 The Interactive Multisensor Analysis Training (IMAT) System: an Evaluation of Airborne Acoustic Mission Course*, (San Diego, CA: GPO, 1998).

and offboard sensors.[26] At present, LosCon can integrate past ASW contact information to provide war-fighters with real-time probabilistic location of target submarines. In the future, LosCon could be part of a tactical decision aid that makes sea shield a reality.

[26] Clark, Vern. *Anti-Submarine Warfare Concept of Operations (Draft 26 April 2004)*.

APPENDIX A: USER'S MANUAL

A. SELECTING A MAP ORIGIN

A latitude and longitude near the area of operations is ideal for a base position. This selection can be changed as desired by the operator, but all the target positions must be updated after doing so. In the graph of target positions, the reference latitude and longitude serve as the map origin, so select a set of coordinates in the southwest corner of the area of operations to keep all information in the first quadrant. Figure 7 shows where to enter the reference coordinates.

	Target1	Target2	Target3
lon_x	8.1294	309.0662	80.0000
lat-y	5.5171	6.3773	80.0000
v_lon	-0.0115	0.0000	0.0000
v_lat	-0.0078	0.0000	0.0000
	Update1	Update2	Update3
Latitude (deg)	35.0000	35.0000	36.0000
Latitude (min)	5.5171	6.3773	20.0000
Longitude (deg)	119.0000	113.0000	118.0000
Longitude (min)	50.0758	42.6998	22.3380

ref_lat (deg)	**35**
ref lon (deg W)	**120**

Figure 7: The map origin is entered as reference latitude and reference longitude in the master spreadsheet.

B. ENTERING CONTACT REPORTS

Contact reports are entered in the master spreadsheet in two formats. A position can be entered directly as a latitude and longitude of the target with associated circularly-symmetric standard deviation error in nautical miles, or a ship can enter its own latitude and longitude combined with the bearing to the contact and standard deviation bearing error as can be seen in Figure 8.

Source	ID	lat (deg)	lat(min)	lon(deg W)	lon(min)	y(nm)	x (nm)	Brg(deg)	Error (deg/nm)	Target#	Time (hrs)	Include(0/1)	Dshock
Knox	1	36	40	119	50	100	8.192	180	5	1	0	1	5E-08
Knox	2	35	45	119	2	45	47.51	225	5	1	0	1	5E-08
Miller	3	35	0	120	0	0	0		200	1	2	2	0.0024

Figure 8: Contact reports are entered into the master spreadsheet.

To enter a bearing contact report, enter the latitude and longitude of the ship that took the bearing in columns three through six. Do not enter anything in columns seven and eight. The north south and east-west coordinates will automatically be calculated in columns seven and eight, respectively. Enter the bearing to the contact in column nine in degrees. Enter the single standard deviation bearing error in column ten. In column eleven, enter on which target the contact report is believed to be. Enter the time of the contact report in column twelve in terms of hours since the beginning of the tracking exercise. Enter a “1” in column thirteen indicating to LosCon that the contact report entered on that row should be included as a bearing rather than a position in the next update of the target’s location. LosCon calculates the dimensionless shock in column fourteen.

To enter a position contact report, enter the latitude and longitude of the position in columns three through six. As before, the north south and east-west coordinates will automatically be calculated in columns seven and eight, respectively. Leave the bearing, column nine, blank. Enter the single standard deviation circularly symmetric error for the position in nautical miles in column ten. In column eleven, enter on which target the contact report is believed to be. Enter the time of the contact report in column twelve, and enter a “2” in column thirteen indicating to LosCon that the contact report entered on that row should be included as a position in the next update of the target’s location. Again, LosCon calculates the dimensionless shock in column fourteen.

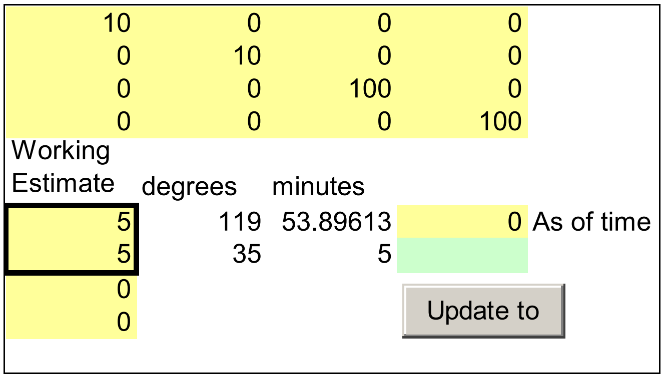
Dimensionless shock should be used as a warning signal. The dimensionless shock can be thought of as how much the program is surprised by the new information the report provides on the submarine’s location. Large dimensionless shock does not mean that a contact report is necessarily false, but a false contact report will probably have a large dimensionless shock. If the past few contact reports were false or had large errors in the same direction, the next true contact report would have a large dimensionless

shock. These are divergence issues. See “E: Divergence and Convergence” for more information.

C. ESTIMATING INITIAL TARGET LOCATION

The operator must estimate an initial position for each submarine. The position must be within a few standard deviations of the first few contact reports for the submarine. An estimate too far from the earliest contact reports may cause the program to return an infinitely large AOU (Area Of Uncertainty) as the program is iterated. This is referred to as divergence. The operator can change the initial position for each submarine at any time, but after selecting an initial position that is feasible compared to the first few contact reports, there is no reason to use a different initial position.

The estimated location and velocity of the target submarine is in the four by one matrix, highlighted in Figure 9. From top to bottom, the cells are east-west position in nautical miles relative to the selected origin, north-south position in nautical miles, east-west velocity component in knots, and north-south velocity component in knots. The four by four matrix, also highlighted, is the covariance matrix ordered in the same way as the location and velocity matrix. The diagonals of the covariance matrix are variances. The latitude and longitude coordinates for the target location are automatically calculated from the values entered for the east-west position and north-south position. If the reference latitude and longitude were changed on the master spreadsheet, the latitude and longitude that is five nautical miles east of the origin and five nautical miles north of the origin would change to correspond. The “Update to” button is used for generating an AOU at some time after the last contact report. See “D: Estimating an AOU.”



10	0	0	0	
0	10	0	0	
0	0	100	0	
0	0	0	100	
Working Estimate	degrees	minutes		
5	119	53.89613	0	As of time
5	35	5		
0				
0			Update to	

Figure 9: An initial estimate for the location, speed, and accuracy of that estimate is entered in spreadsheet, "Target1."

An initial estimate for the location of each submarine must be entered in LosCon. This value, if reasonable, will be changed to the optimal initial estimate in the course of iteration. The next ten figures will show how to iterate the extended Kalman filter until it converges on an optimal initial estimate. As demonstrated in Figure 10, an initial estimate for the location of each target submarine should be entered in the two cells immediately below "Working Estimate." These are east-west and north-south coordinates in nautical miles. The origin is the one selected on the "Master" spreadsheet. See "A: Selecting a Map Origin." The latitude and longitude, which correspond to the estimated location, will appear in degrees and minutes next to the two entries.

0	0	0	0	
0	0	0	0	
0	0	0	0	
0	0	0	0	
Working				
Estimate	degrees	minutes		
0	120	0		As of time
0	35	0		
0			Update to	
0				
Initial				
Location				
Estimate	degrees	minutes		
	120	0	As of time 0.	
	35	0		

Figure 10: An initial estimate is entered in spreadsheet "Target1" as (x = 0 nm, y = 0 nm) and a velocity of zero. The position corresponds to (35 ˚N, 120 ˚W), the selected map origin.

Use the first few bearing or position reports to find the optimal initial estimate for the location of a submarine. Figure 11 shows two sample bearing contact reports. Based on these two measurements, the initial position estimate will be iterated until it converges or diverges after pressing the "Update1" button repeatedly. Figures 12 through 15 show the values of the position estimates based on the contact reports from Figure 11. That position estimate is iterated until it converges. See "D: Divergence and Convergence."

Source	ID	lat (deg)	lat(min)	lon(deg W)	lon(min)	y(nm)	x (nm)	Brg(deg)	Sig(deg)	Target#	Time (hrs)	Include(0/1)	Dshock
Knox	1	36	40	119	50	100	8.192	180	5	1	0	1	9E-06
Knox	2	35	45	119	2	45	47.51	225	5	1	0	1	8E-06

Figure 11: The two bearing contact reports are entered for "Target #1" on the "Master" spreadsheet.

	Target1	Target2	Target3
lon_x	0.0000	0.0000	0.0000
lat-y	0.0000	0.0000	0.0000
v_lon	0.0000	0.0000	0.0000
v_lat	0.0000	0.0000	0.0000
	Update1	Update2	Update3
Latitude (deg)	35.0000	35.0000	35.0000
Latitude (min)	0.0000	0.0000	0.0000
Longitude (deg)	120.0000	120.0000	120.0000
Longitude (min)	0.0000	0.0000	0.0000
ref_lat (deg)	**35**		
ref lon (deg W)	**120**		

Figure 12: The estimated location for "Target #1" on the "Master" spreadsheet after entering an estimate is the same as was entered on the "Target1" spreadsheet.

	Target1	Target2	Target3
lon_x	10.3994	0.0000	0.0000
lat-y	10.1541	0.0000	0.0000
v_lon	0.0000	0.0000	0.0000
v_lat	0.0000	0.0000	0.0000
	Update1	Update2	Update3
Latitude (deg)	35.0000	35.0000	35.0000
Latitude (min)	10.1541	0.0000	0.0000
Longitude (deg)	119.0000	120.0000	120.0000
Longitude (min)	47.3047	0.0000	0.0000
ref_lat (deg)	**35**		
ref lon (deg W)	**120**		

Figure 13: The estimated location for "Target #1" changes after the "Update1" button is pressed once.

	Target1	Target2	Target3
lon_x	7.9653	0.0000	0.0000
lat-y	4.8648	0.0000	0.0000
v_lon	0.0000	0.0000	0.0000
v_lat	0.0000	0.0000	0.0000
	Update1	Update2	Update3
Latitude (deg)	35.0000	35.0000	35.0000
Latitude (min)	4.8648	0.0000	0.0000
Longitude (deg)	119.0000	120.0000	120.0000
Longitude (min)	50.2761	0.0000	0.0000
ref_lat (deg)	**35**		
ref lon (deg W)	**120**		

Figure 14: The estimated location for "Target #1" continues to change the second time the "Update1" button is pressed.

	Target1	Target2	Target3
lon_x	8.1915	0.0000	0.0000
lat-y	5.5655	0.0000	0.0000
v_lon	0.0000	0.0000	0.0000
v_lat	0.0000	0.0000	0.0000
	Update1	Update2	Update3
Latitude (deg)	35.0000	35.0000	35.0000
Latitude (min)	5.5655	0.0000	0.0000
Longitude (deg)	119.0000	120.0000	120.0000
Longitude (min)	50.0000	0.0000	0.0000
ref_lat (deg)	**35**		
ref lon (deg W)	**120**		

Figure 15: After the "Update1" button is pressed nine times the estimated target location converges at (35° 5.6' N, 119° 50' W).

After an optimal initial estimate for the location of a submarine is found, that value needs to be saved for future use. Every time a new contact report on that submarine is entered, the optimal initial estimate should be re-entered as well. Figure 16 shows the two position entries from the optimal initial estimate being saved.

92.12218	92.332	0	0		
92.332	156.8761	0	0		
0	0	100	0		
0	0	0	100		
Working Estimate	degrees	minutes			
8.191517	119	50	0	As of time	
5.565505	35	5.565505			
0			Update to		
0					
Initial Location Estimate	degrees	minutes			
8.191517	119	50	As of time 0.		
5.565505	35	5.565505			

Figure 16: The optimized initial estimate in spreadsheet "Target1" is copied from the "Working Estimate" cells to the "Initial Location Estimate" cells for later use.

A new contact report for "Target #1" is shown in Figure 17. The estimate for the location of the submarine no longer needs to be iterated, so the "Update1" button should be pressed only once. Figures 18 and 19 show how that new contact report changes the estimate location at the time of the third contact on the "Master" and "Target1" spreadsheets.

Source	ID	lat (deg)	lat(min)	lon(deg W)	lon(min)	y(nm)	x (nm)	Brg(deg)	Sig(deg)	Target#	Time (hrs)	Include(0/1)	Dshock
Knox	1	36	40	119	50	100	8.192	180	5	1	0	1	2E-24
Knox	2	35	45	119	2	45	47.51	225	5	1	0	1	4E-24
miller	3	35	0	120	0	0	0		200	1	2	2	

Figure 17: A new "Target #1" contact report for a time two hours later is entered.

	Target1	Target2	Target3
lon_x	8.1422	0.0000	0.0000
lat-y	5.5131	0.0000	0.0000
v_lon	-0.0051	0.0000	0.0000
v_lat	-0.0034	0.0000	0.0000
	Update1	Update2	Update3
Latitude (deg)	35.0000	35.0000	35.0000
Latitude (min)	5.5131	0.0000	0.0000
Longitude (deg)	119.0000	120.0000	120.0000
Longitude (min)	50.0602	0.0000	0.0000
ref_lat (deg)	**35**		
ref lon (deg W)	**120**		

Figure 18: The estimated location of the submarine is shown on spreadsheet "Master" after entering a third contact report and pressing "Update1" once.

178.6139	91.36253	24.88002	-0.05708	
91.36253	242.689	-0.05708	24.83998	
24.88002	-0.05708	50.00123	3.57E-05	
-0.05708	24.83998	3.57E-05	50.00125	
Working Estimate	degrees	minutes		
8.14223	119	50.06017	2	As of time
5.513068	35	5.513068		
-0.00509			Update to	
-0.00344				
Initial Location Estimate	degrees	minutes		
8.191517	119	50	As of time 0.	
5.565505	35	5.565505		

Figure 19: The estimated location of the submarine is also shown on spreadsheet "Target1" after entering a third contact report and pressing "Update1" once.

The time in hours shown in the two cells above the "Update to" button is the time of the last contact report and is the time for which the "Working Estimate" of submarine location is valid. The "Working Estimate" may not be a good estimate for time zero, so as new contacts are entered on the "Master"' spreadsheet, the values saved in "Initial

Location Estimate" should be entered in the two cells in the box immediately below "Working Estimate" before pressing the "Update" button.

D. ESTIMATING AN AOU

On each target-specific spreadsheet, the highlighted cell above the button, "Update to" contains the time of the AOU currently being displayed. The AOU at the time of the last contact report is automatically calculated as shown in Figure 20. The center of the AOU is the position shown under "Working Estimate." The orientation in degrees is the bearing of the AOU's major axis. The value under "major(nm)" is the major axis's length in nautical miles. Likewise, the value under "minor(nm)" is the minor axis's length in nautical miles. The minor axis is perpendicular to the major axis, and thus it has an orientation ninety degrees off the orientation listed.

To generate an AOU for a time after the time of the last contact report, do not directly change the number in the cell which displays the time of the last contact report. That will generate an inaccurate result, because the other numbers used in the calculation will not have been updated. Entering the time desired in hours in the cell above the "Update to" button, selecting a different cell, and then clicking on "Update to" will move the number entered up one row and recalculate that submarine's AOU for the desired time. Figure 20 and Figure 21 show an AOU being updated to a future time.

178.7752	91.66242	24.87992	-0.05727	
91.66242	243.2563	-0.05727	24.83963	
24.87992	-0.05727	50.00123	3.58E-05	
-0.05727	24.83963	3.58E-05	50.00125	
	degrees	minutes		
8.14684	119	50.05454	2	As of time
5.526056	35	5.526056		
-0.00519				
-0.00352			Update to	

AOU		
major(nm)	minor(nm)	orient(deg)
70.220554	42.679945	35.310811

Figure 20: Changing the time generates an updated AOU.

278.6576	91.60621	24.99781	-0.00105	
91.60621	343.0992	-0.00105	24.99707	
24.99781	-0.00105	50	1.2E-08	
-0.00105	24.99707	1.2E-08	50	
	degrees	minutes		
8.144295	119	50.05765	4	As of time
5.524329	35	5.524329		
-9.5E-05			Update to	
-6.4E-05				

AOU		
major(nm)	minor(nm)	orient(deg)
80.794647	58.483616	35.310811

Figure 21: The matrix is recalculated after the "Update to" button is clicked.

E. DIVERGENCE AND CONVERGENCE

When LosCon is being used optimally, the iteration of the filter (repeatedly pressing the "Update" button) will cause convergence. This means that with each iteration, the amount of change to the center point of the AOU will decrease. Iteration is desirable, because it increases the accuracy of the program by minimizing the effect of the initial estimate of the target's position.[27] The goal is to generate an "Initial Location Estimate" by iterating the filter with the first few contact reports. Those values should be used for the initial estimate every time a new contact report is entered in LosCon. See "C: Estimating Initial Target Locations" for details on when and how to iterate the estimated target location.

Divergence refers to the event where the extended Kalman filter fails to correct errors in the appropriate direction and instead compounds them by moving the target location estimate farther and farther from the true location as the filter is iterated. Divergence can be caused by a poor initial target estimate or a series of false contact reports that are inputted as true. When iteration does not result in decreasing changes in the location of the center of the AOU, divergence has occurred.

If divergence occurs, a new initial estimate for the location and velocity of the target must be entered. LosCon's working estimate for the location of the submarine is a

[27] Kerr, Thomas H.. "Streamlining Measurement Iteration for EKF Target Tracking," *IEEE Transactions on Aerospace and Electronic Systems* 27, no. 2 (March 1991): 408-421.

product of the last update. If the program has just diverged, that working estimate is probably an extremely bad initial estimate. Changing the initial estimate alone may be sufficient to reacquire a convergence.

If entering a new initial estimate alone is not sufficient, again enter a feasible initial estimate and select a different set of early contact reports to use in the next attempt to find a converging estimate for the location of the target submarine at time zero. Recall that LosCon allows reports to be included and left out with a simple toggle switch at any time. Trying a variety of sets of reports and reentering a feasible initial estimate each time, should result in converging output.

F: OPERATION SUMMARY

LosCon's iterated extended Kalman filter, which generates the center coordinate of the AOU, works in Cartesian coordinates. The operator must enter a reference latitude and longitude point for the program to minimize the error associated with converting approximately spherical coordinates to locally-flat space. LosCon automatically converts the latitude and longitude coordinates entered by the operator in contact reports to east-west and north-south coordinates in nautical miles referenced to the origin chosen by the operator. The outputs are presented in Cartesian coordinates and in latitude and longitude. See "A: Selecting a Map Origin."

Contact reports can be bearings or positions. The contact report database remains available to the operator to change and or update as desired. Contact reports ideally consist of the raw data: the latitude and longitude of the observer combined with a bearing on the target and an estimated standard deviation bearing error at a given time. If the contact report is instead a position derived from TMA (Target Motion Analysis), LosCon will accept a target latitude and longitude combined with that position's standard deviation circularly symmetric error in nautical miles and the time of the report. See "B: Entering Contact Reports."

On the subsidiary spreadsheets assigned to each potential target, the operator must enter an initial estimate for the enemy submarines' locations at time zero. A good estimate is a latitude and longitude near the search assets but not exactly the same. If a submarine is found it will be relatively near the searchers, so that initial estimate is logical and unlikely

to make the search problem diverge. If the first estimate results in divergence, the operator can simply try a new one a few seconds later. The initial estimate includes two velocity components as well as location; these velocity estimates do not affect convergence and should usually be set to zero. See "C: Estimating Initial Target Location" and "E: Divergence and Convergence."

The AOUs generated on the target specific spreadsheets correspond to each individual target. The centers of the AOUs are updated on the master spreadsheet as well. AOUs can also be calculated for future times. See "D: Estimating an AOU."

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APPENDIX B: VISUAL BASIC CODE

A. MASTER CALCULATIONS

1. Description of Code

LosCon's code is after the MTST (Maneuvering Target Statistical Tracker) program by Professor Alan Washburn of the Naval Postgraduate School. Daniel H.Wagner designed the first MTST program in 1980. A copy of LosCon.xls can be obtained from Prof. Washburn (email: awashburn@nps.edu).

The following code has been modified to include three distinct targets, false contact reports, position and bearing target types, and latitude and longitude inputs vice Cartesian coordinates. Changing the default single standard deviation for the root mean square of the target submarines' velocities and changing the default long-term average velocity of the submarines can be done in Kmove subroutine.

2. Initialization and Cartesian Conversion

a. Initialization and Global Variables

```
Option Base 1
Option Explicit
Dim Sigma, Mu
```

b. Latitude to Cartesian Coordinate Conversion

```
Function Latdeg(y, latref, lonref)
Dim latitude
latitude = latref + y / 60
Latdeg = Int(latitude)
End Function

Function Latmin(y, latref, lonref)
Dim latitude
latitude = latref + y / 60
Latmin = (latitude - Int(latitude)) * 60
```

```
End Function

Public Function yLat(Latdeg, Latmin, latref, lonref)
Dim latitude
latitude = Latdeg + Latmin / 60yLat = (latitude - latref) * 60
End Function
```

c. Longitude to Cartesian Coordinate Conversion

```
Function Londeg(x, latref, lonref)
Dim longitude, fac
fac = 60 * Cos(Atn(1) * latref / 45)
longitude = lonref - x / fac
Londeg = Int(longitude)
End Function

Function Lonmin(x, latref, lonref)
Dim longitude, fac
fac = 60 * Cos(Atn(1) * latref / 45)
longitude = lonref - x / fac
Lonmin = (longitude - Int(longitude)) * 60
End Function

Public Function xLon(Londeg, Lonmin, latref, lonref)
'longitudes are in degrees west
'xLon is positive if longitude is east of the reference.
Dim longitude, fac
fac = 60 * Cos(Atn(1) * latref / 45)
longitude = Londeg + Lonmin / 60
xLon = (lonref - longitude) * fac
End Function
```

3. Updating Target Submarines

```
Public Sub Update(target)
Dim Minputs, Mu, Sigma
Dim nrows&, ncols&, sht$
Select Case target
Case 1
   sht = "Target1"
   With Worksheets(sht)
      Sigma = .Range("MAT1").Value
      Mu = .Range("muin").Value
   End With
Case 2
   sht = "Target2"
   With Worksheets(sht)
```

```
      Sigma = .Range("MAT1").Value
      Mu = .Range("muin").Value
   End With
Case 3
   sht = "Target3"
   With Worksheets(sht)
      Sigma = .Range("MAT1").Value
      Mu = .Range("muin").Value
   End With
Case Else
   Exit Sub
End Select
With Range("MasterIn")
   Minputs = .Value
   nrows = .Rows.Count
   ncols = .Columns.Count
      With .Interior
      .ColorIndex = 35
      .Pattern = xlSolid
      End With
End With
Dim i&, j&, k&, t#
For i = 1 To 4
   For j = 1 To 4
      Sigma(i, j) = 0
   Next j
Next i
Sigma(1, 1) = 100000: Sigma(2, 2) = 100000
Sigma(3, 3) = 100: Sigma(4, 4) = 100
t = 0
For k = 1 To nrows
   'If include doesn't  = 0
   If Minputs(k, 12) > 0 Then
      'If this is the target you are processing
     If Minputs(k, 10) = target Then
       Call KMove(Mu, Sigma, Minputs(k, 11) - t)
       t = Minputs(k, 11)
       If Minputs(k, 12) = 1 Then
          Call KMeas(Mu, Sigma, Minputs(k, 7), Minputs(k, 6), Minputs(k, 8),
Minputs(k, 9), Minputs(k, 13))
       Else
          Call KMeasPos(Mu, Sigma, Minputs(k, 7), Minputs(k, 6), Minputs(k, 9),
Minputs(k, 13))
       End If
       Range("MasterIn")(k, 13).Value = Minputs(k, 13)
      End If
```

```
    'For not included rows
    Else
        Range("MasterIn")(k, 13).Value = ""
    End If
Next k
With Worksheets(sht)
     .Range("MAT1").Value = Sigma
     .Range("muin").Value = Mu
     .Range("CurTime")(1, 1).Value = t
End With
'Worksheets("Master").Range(sht).Value = Mu
End Sub
```

4. Bearing Measurements

```
Sub KMeas(Mu, Sigma, x, y, theta, sig, dshock)
Dim dx#, dy#, ran#, ang#, fac#, H(1, 4) As Double
Dim Shock#, r#, e1, e2, denom, KGain(4, 1) As Double
Dim lat#, lon#, latmu#, lonmu#, latref#, lonref#
Dim beta#, conv#
conv = Atn(1) / 45
latref = CDbl(Worksheets("master").Range("p13").Value)
lonref = Worksheets("master").Range("p14").Value
conv = 1 / (60 * Cos(latref * conv))
lat = (latref + y / 60)
lon = (lonref - conv * x)
latmu = (latref + Mu(2, 1) / 60)
lonmu = (lonref - conv * Mu(1, 1))
dx = Mu(1, 1) - x: dy = Mu(2, 1) - y
ran = dx * dx + dy * dy
If ran = 0 Then Exit Sub
r = (sig * conv) ^ 2
' following two statements replaced by bearing() call 4/21/04
'ang = Atn2(dy, dx)
'Shock = theta * conv - ang
ang = bearing(lat, lonmu, latmu, lon) 'note lon reversal
Shock = (theta - ang) * conv
If Shock > 4 * Atn(1) Then
    Shock = Shock - 8 * conv
ElseIf Shock < -4 * Atn(1) Then
    Shock = Shock + 8 * conv
End If
H(1, 1) = dy / ran: H(1, 2) = -dx / ran
H(1, 3) = 0: H(1, 4) = 0
e1 = Application.WorksheetFunction.Transpose(H)
e2 = Application.WorksheetFunction.MMult(Sigma, e1)
denom = Application.WorksheetFunction.MMult(H, e2)
```

```
dshock = Shock * Shock / (denom(1) + r)
If dshock > 1000 Then
   Exit Sub 'but dshock is still returned
End If
'update mu and sigma
fac = (denom(1) + r) ^ (-1)
' can't figure out how to make a function return an array, so use subs
' last argument in subs is the output
Call matscal(e2, fac, KGain)
Call matscal(KGain, Shock, e1)
Call matadd(Mu, e1, Mu)
e1 = Application.WorksheetFunction.MMult(KGain, H)
e2 = Application.WorksheetFunction.MMult(e1, Sigma)
Call matdif(Sigma, e2, Sigma)
'Debug.Print dx; dy; ran
End Sub
```

5. Position Measurements

```
Sub KMeasPos(Mu, Sigma, x, y, sig, dshock)
'input is a target location, not a bearing
'x and y are the measured target location
'sig is the standard deviation of x and of y, in compatible units
Dim q#, H(2, 4) As Double
Dim Shock(2, 1) As Double, e1, e2, denom, KGain
Shock(1, 1) = x - Mu(1, 1): Shock(2, 1) = y - Mu(2, 1)
q = sig * sig
H(1, 1) = 1: H(1, 2) = 0: H(1, 3) = 0: H(1, 4) = 0
H(2, 1) = 0: H(2, 2) = 1: H(2, 3) = 0: H(2, 4) = 0
e1 = Application.WorksheetFunction.Transpose(H)
e2 = Application.WorksheetFunction.MMult(Sigma, e1)
denom = Application.WorksheetFunction.MMult(H, e2)
denom(1, 1) = denom(1, 1) + q: denom(2, 2) = denom(2, 2) + q
denom = Application.WorksheetFunction.MInverse(denom)
e1 = Application.WorksheetFunction.MMult(denom, Shock)
dshock = Shock(1, 1) * e1(1, 1) + Shock(2, 1) * e1(2, 1)
If dshock > 1000 Then
   Exit Sub 'but dshock is still returned
End If
KGain = Application.WorksheetFunction.MMult(e2, denom)
e1 = Application.WorksheetFunction.MMult(KGain, Shock)
' can't figure out how to make a function return an array, so use subs
' last argument in subs is the output
Call matadd(Mu, e1, Mu) 'Mu is now updated
e1 = Application.WorksheetFunction.MMult(KGain, H)
e2 = Application.WorksheetFunction.MMult(e1, Sigma)
Call matdif(Sigma, e2, Sigma) 'Sigma is now updated
```

```
End Sub
```

6. MTST Movement Subroutine

```
Sub KMove(Mu, Sigma, t As Double)
Dim s#, beta#, b1#, b2#, c#, c1#, c2#, c3#
s = 10: beta = 2
's is rms speed (kt), 1/beta is the relaxation time (hr)
'To change s the root mean square speed of the target submarines
'to potentially different rms speeds for each, replace the line s=10 with
'Dim s1#, s2#, s3#
's1 = Worksheets("Target1").Range("name of cell").Value
's2 = Worksheets("Target2").Range("name of cell").Value
's3 = Worksheets("Target3").Range("name of cell").Value
'where name of cell is a2 or h32 for example.
'Be sure to keep the quotation marks when choosing a cell in which to
'input the rms speeds.
Dim matin, matout, e1, e2, e3, e4
Dim i&, j&, k&
'Dim e1(4, 4) As Double, e2(4, 4) As Double, e3(4, 4) As Double, e4(4, 4) As
Double
Dim Phi(4, 4) As Double, q(4, 4) As Double
'matin = mat.Value
'matout = Application.WorksheetFunction.MInverse(matin)
For i = 1 To 4
   For j = 1 To 4
      Phi(i, j) = 0: q(i, j) = 0
   Next j
Next i
If beta > 0 Then
   If t * beta < 700 Then
      b2 = Exp(-t * beta)
   Else
      b2 = 0
   End If
   b1 = (1 - b2) / beta
   c = 0.5 * s * s
   c1 = c * (t + t - (3 - 4 * b2 + b2 * b2) / beta) / beta
   c2 = c * beta * b1 * b1
   c3 = c * (1 - b2 * b2)
Else
   b2 = 1
   b1 = t
   c1 = 0
   c2 = 0
   c3 = 0
End If
```

```
Phi(1, 1) = 1: Phi(2, 2) = 1
Phi(3, 3) = b2: Phi(4, 4) = b2
Phi(1, 3) = b1: Phi(2, 4) = b1
q(1, 1) = c1: q(2, 2) = c1
q(3, 3) = c3: q(4, 4) = c3
q(1, 3) = c2: q(2, 4) = c2: q(3, 1) = c2: q(4, 2) = c2
e1 = Application.WorksheetFunction.MMult(Phi, Sigma)
e2 = Application.WorksheetFunction.Transpose(Phi)
e3 = Application.WorksheetFunction.MMult(e1, e2)
Mu = Application.WorksheetFunction.MMult(Phi, Mu)
Call matadd(e3, q, Sigma)
'Defining Mu_W (long-term average velocity) in terms
'of horizontal east-west velocity (HorV)
'and in terms of vertical north-south velocity (VerV)
Dim HorV#, VerV#
'To change the long-term average velocity from zero to
'an input value in cells s13 and s14, comment the next two lines and uncomment
'the lines defining HorV and VerV as called values.
HorV = 0
VerV = 0
'HorV = Worksheets("master").Range("s13").Value
'VerV = Worksheets("master").Range("s14").Value
'Adding Mu_W for the long-term average velocity of the target
Mu(3, 1) = Mu(3, 1) + HorV * (1 - b2)
Mu(4, 1) = Mu(4, 1) + VerV * (1 - b2)
End Sub
```

7. Orientation of Compass

```
Public Function Atn2(x, y)
'-pi<=atn2<pi
'counterclockwise from East in radians
'Atn2(y,x) is clockwise from North in radians
Const pi2 = 1.570796326794
If x > 0 Then
  Atn2 = Atn(y / x)
ElseIf x < 0 Then
  If y > 0 Then
    Atn2 = Atn(y / x) + pi2 + pi2
  Else
    Atn2 = Atn(y / x) - pi2 - pi2
  End If
ElseIf y > 0 Then
  Atn2 = pi2
Else
  Atn2 = -pi2
End If
```

```
End Function
```

8. Course and Bearing Functions

```
Function bearing(lat1, lon1, lat2, lon2)
'course in degrees from N clockwise, inputs in degrees, Wlon<0
Dim conv#, x1#, x2#, x3#, x4#
conv = Atn(1) / 45
x1 = lat1 * conv: x2 = lon1 * conv: x3 = lat2 * conv: x4 = lon2 * conv
bearing = course(x1, x2, x3, x4) / conv
End Function

Function course(lat1, lon1, lat2, lon2)
'course in radians from N clockwise to get from 1 to 2
Dim cd#, dlon#, c1#, c2#, s1#, s2#
c1 = Cos(lat1)
If c1 <= 0 Then 'north or south pole
   If lat1 > 0 Then
      course = 4 * Atn(1)
   Else
      course = 0
   End If
Else
   dlon = lon2 - lon1
   c2 = Cos(lat2): s1 = Sin(lat1): s2 = Sin(lat2)
   cd = s1 * s2 + c1 * c2 * Cos(dlon)
   course = Atn2((s2 - s1 * cd) / c1, c2 * Sin(dlon))
   If course < 0 Then
      course = course + 8 * Atn(1)
   End If
End If
End Function
```

9. Matrix Algebra

```
Sub matadd(A, B, c)
Dim i&, j&
For i = 1 To UBound(A, 1)
For j = 1 To UBound(A, 2)
c(i, j) = A(i, j) + B(i, j)
Next j
Next i
End Sub

Sub matdif(A, B, c)
Dim i&, j&
For i = 1 To UBound(A, 1)
```

```
For j = 1 To UBound(A, 2)
c(i, j) = A(i, j) - B(i, j)
Next j
Next i
End Sub

Sub matscal(A, B, c)
Dim i&, j&
For i = 1 To UBound(A, 1)
For j = 1 To UBound(A, 2)
c(i, j) = A(i, j) * B
Next j
Next i
End Sub
```

B. UPDATING AOU TO FUTURE TIME T

```
Private Sub UpdateTo_Click()
With Worksheets("target1")
   Sigma = .Range("a1:d4").Value
   Mu = .Range("a6:a9").Value
   told = .Range("CurTime")(1, 1).Value
   tnew = .Range("CurTime")(2, 1).Value
   If told < tnew Then
      Call KMove(Mu, Sigma, tnew - told)
      .Range("a1:d4").Value = Sigma
      .Range("a6:a9").Value = Mu
      .Range("CurTime")(1, 1).Value = tnew
      .Range("CurTime")(2, 1).Value = ""
   End If
End With
End Sub
```

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